S.P. 2-L

For $\mathbf{A}(\mathbf{r})=r \hat{\mathbf{a}}_{r}+\hat{\mathbf{a}}_{\theta}+r \cos \phi \hat{\mathbf{a}}_{\phi}$, find the value of $\oint_{S} \mathbf{A} \cdot \mathbf{d s}$ over the 3-D "Pac-Man" surface shown below, which is a unit sphere, minus the $45^{\circ}$ "mouth" the top. The volume enclosed by $S$ includes all points $0 \leq r \leq 1, \frac{\pi}{4} \leq \theta \leq \pi, 0 \leq \phi \leq 2 \pi$. Find the value of this flux integral by:
a) Evaluating the surface integral directly
b) Use of the divergence theorem


